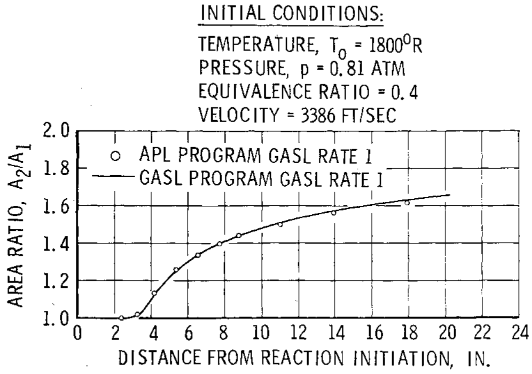

Fig. 3 Comparison of APL and GASL computer programs for constant pressure H_2-air reaction with identical inputs and rates.

To further check the two computer programs, identical rates and gas properties were used with both programs to learn whether they would produce identical results. A mathematical analysis showed that the programs were mathematically very similar.[4] The results in Fig. 3 indicate that they will produce essentially identical results.

Thus, any substantial variation in the recombination zone histories results from differences in the selected chemical reaction rates. Each rate is determined by theoretical calculations or experimental measurements; its accuracy is affected by initial assumptions or inaccuracy in measurement, as the case may be.

The question remains, how closely can the recombination history be predicted by using existing selected rates in machine computations? Some experimental work has been done using a normal-shock standing-wave at the entrance to a tube to initiate the chemical reactions and measuring the pressure effects along the constant-area-tube wall. Preliminary results have been reported in Ref. 4, and the work is still in progress.

The conclusions drawn are as follows

1) The APL and GASL computer programs produce identical results for identical input conditions for H_2-air reactions.

2) The differences among the various reaction rate constants used in the two programs are within the realm of existing theoretical and experimental knowledge. However, the resulting differences in heat release rates in a hypersonic air-breathing propulsion system where reaction chemistry is the controlling factor may be unsatisfactory with respect to performance definition. Hence, more accurately determined rate constants are needed.

References

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Comments on "Moment of Inertia and Damping of Liquids in Baffled Cylindrical Tanks"

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Nomenclature

a	= radius of container
b	= $a - w$
d	= distance between baffles
h	= height of container
h^*	= equivalent height of liquid above baffle carried along with baffle
I_s	= moment of inertia of solidified liquid
I_e	= effective moment of inertia of liquid
I_B^*, I_B^{**}	= moment of inertia of liquid trapped between baffles
k	= $(a - w)/a$
m^*, m^{**}	= mass of liquid trapped between baffles
s	= baffle coordinate
w	= baffle width
w_0	= velocity of liquid
ϵ_n	= zeros of first derivation of Bessel function of first kind and first order $[J_1'(\epsilon_n) = 0]$
ρ	= mass density of liquid
$\zeta = \xi + i\eta$	= elliptic coordinates
φ	= polar angle
θ_0	= excitation amplitude about y axis
Φ	= velocity potential
ψ	= stream function
Ψ	= complex stream function

Subscripts

b_1	= for baffle 1
b_d, b_{2d}	= for baffle at d and $2d$, respectively
r	= rigid
swb	= for smooth wall for radius b

DODGE and Kana[1] gave results for the moment of inertia of a liquid in a completely filled cylindrical container of circular cross section. The analytically computed moment of inertia (I) for ideal liquid[2] and smooth container walls agrees very well with the experimental results, which, because of internal and wall friction, exhibit only slightly larger values for I. Their note did not give any analytical approximation for I for the liquid in a container with ring baffles. The ring baffles will essentially take additional liquid along as they are moving, and thus increase the effective I of the liquid. To compare their experimental result with the following theoretical results, an approximation for the effective mass of liquid connected with the motion of each baffle is presented.

For very closely spaced baffles throughout the container, it would seem reasonable to assume that the liquid is decomposed into two domains: 1) an annular cylinder of inner radius (a-w) and outer radius, a, where w is the baffle width, and 2) a circular cylinder with radius $b = a - w$. In domain 1. the liquid can, for a distance between baffles $d \leq \pi w/2$, be approximated by a rigid body, while in domain 2 the liquid is assumed to behave like that in a container of smaller radius $b = a - w$. The liquid trapped between the baffles has a moment of inertia

$$I_B^* = m^*a^2[\tfrac{1}{12}(h/a)^2 + (1 + k^2)/4]$$

where $k = (a - w)/a$ represents the ratio of the inner radius $b = a - w$ to the outer radius a, and m^* is the mass of the trapped liquid, i.e., $m^* = \pi\rho h w(2a - w)$.

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The effective moment of the liquid in a circular cylinder with closely spaced baffles therefore is given by

$$I_e = \pi\rho h b^2 \left\{ \frac{h^2}{12} - \frac{b^2}{4} + 4\sum_{n=1}^{\infty} \frac{b^2\left[\dfrac{2\tanh(\epsilon_n h/2b)}{\epsilon_n h/2b} - 1\right]}{\epsilon_n^2(\epsilon_n^2 - 1)} \right\} + \rho\pi h w(2a - w)\left[\frac{h^2}{12} - \frac{2a^2 - w^2}{4}\right] \quad (1)$$

For $w \to 0$, this expression approaches the analytical result presented in Fig. 2 of Ref. 1. For such a close spacing of baffles and $w = a$, i.e., baffles that are circular plates subdividing the container into circular slices, $I_e = I_s$ (for solidified liquid). The value of Eq. (1) represents an upper limit of I_e for the liquid.

For a wider spacing of baffles, of course, the situation is much more complex, and an approximation is presented in the following. The basic idea is the determination of the amount of liquid that is carried along by the baffle during its motion. To find this mass approximately one considers a two-dimensional flat plate of width w. If the pressure on the baffle is determined, then the total force on the baffle can be given and the effective mass can be determined by dividing this force by the acceleration of the baffle.

If $w \ll a$, then the velocity given by the smooth wall solution at the location of the baffle can for all practical purposes be considered constant over w.

The disturbance potential created by the baffle/unit length can be approximated by the potential of a flat plate in a uniform flow. Adding the image of the baffle to the configuration makes the container wall a streamline. Now one is able to solve the problem by the application of complex functions.[3] The complex stream function is

$$\Psi = \Phi + i\psi = iw_0 w \sinh\zeta \quad (2)$$

where

$$\zeta = \xi + i\eta \quad \text{and} \quad z = x + iy = w\cosh\zeta$$

$$0 \leq \xi < \infty \quad \text{and} \quad -\pi \leq \eta \leq +\pi$$

(at the container wall $\eta = \pm\pi/2$, and of the baffle, it is $\xi = 0$). The potential function is

$$\Phi = -w_0 w \cosh\xi \sin\eta \quad (3)$$

and the stream function yields

$$\psi = w_0 w \cos\eta \sinh\xi \quad (4)$$

Subtracting the potential function of the parallel stream, one obtains the disturbance potential

$$\Phi^* = w_0 w e^{-\xi} \sin\eta \quad (5)$$

From the unsteady Bernoulli equation one obtains finally by neglecting higher order terms for the disturbance pressure[3] the value

$$p^* = \mp\rho\dot{w}_0(w^2 - s^2)^{1/2} \quad (6)$$

where s is the baffle coordinate, being zero at the root of the baffle and $s = w$ at the outer edge of the baffle.

It can be seen that the stationary (quadratic) term in the Bernoulli equation would present only a small contribution proportional to the pitching excitation amplitude θ_0. The ideal flow around a flat plate yields negative infinite pressures at the edge of the plate, indicating that the flow must separate at these locations. The solution of Kirchhoff[4] considering an infinite dead zone behind the plate yields a pressure/unit length of the plate in the amount of about $p/2w = 0.88\,\rho w_0^2/2$, which, according to measurements of Prandtl[5], exhibits quite a deviation, namely, the 0.88 becomes 2.0 because of the closing of the dead zone behind the plate. The approximate effective moment of inertia caused by this part would result in an expression $I_e/I_s \approx (0.08 - 0.16)\theta_0$, which, for all practical purposes, can be neglected in comparison with the instationary part.

To determine now the effective mass of the liquid carried along with a baffle in motion, it is for the force on a strip of the baffle

$$2p^*dA = 2\rho\dot{w}_0(w^2 - s^2)^{1/2}dA \quad (7)$$

where $dA = 1\cdot ds$, which is the area of the baffle of circumferential length unity and width ds. The effective mass per circumferential length and the corresponding liquid height h^* above the baffle are

$$\mu_e = \pi\rho w^2/2 \qquad h^* = \pi w/2 \quad (8)$$

from which one can conclude that for the baffle spacing $d > \pi w/2$, the preceding method should be used and that for $d \leq \pi w/2$, the previously mentioned trapped liquid method should be employed. For the container given in the note[1], $h/a = 2$, $d/a = 0.4$ and $w/a = 0.155$. Then for the liquid near one baffle in the axis of rotation,

$$\frac{I_e^{(1)}}{I_s} = \frac{\dfrac{\pi}{2}\rho w^2 a^3 \displaystyle\int_0^{2\pi}\cos^2\varphi d\varphi}{ma^2[\frac{1}{12}(h/a)^2 + \frac{1}{4}]} = \frac{\pi\left(\dfrac{w}{a}\right)^2}{2(h/a)[\frac{1}{12}(h/a)^2 + \frac{1}{4}]} \quad (9)$$

which yields the value 0.033. Adding this value to the smooth wall result for radius b of 0.21 yields for the given container with one baffle,

$$I_e^{(1)}/I_r = 0.243 \quad (10)$$

a value that is a little smaller than the measured one, but agrees quite well. For the container with three baffles the effective moment of inertia can be obtained similarly. The distance of the upper and lower baffle from the axis of rotation is $(a^2 + d^2)^{1/2}$, or $a^2[1 + (d/a)^2]^{1/2}$. The effect of the previous baffle has to be multiplied by this factor to give the approximate contribution of the upper or lower baffle. It is therefore

$$I_e^{(3)}/I_r = (I_e/I_r)_{swb} + (I_e/I_r)_{b_1} + 2(I_e/I_r)_{bd} \quad (11)$$

which yields for the given container

$$I_e^{(3)}/I_r = 0.21 + 0.033 + 2(0.0385) \approx 0.32 \quad (12)$$

Again this value is smaller than the experimental value. If one would treat the liquid between the baffles as completely trapped one would obtain with the trapped mass $m^{**} = 2\pi\rho d(2a - w)$ and its moment of inertia

$$I_B^{**} = m^*a^2[\tfrac{1}{3}(d/a)^2 + (1 + k^2)\,4] \quad (13)$$

Table 1 Comparison of theoretical approximations and experiment

$n^a = 0$		$n = 1$		$n = 3$		$n = 5$	
Theory	Experiment	Theory	Experiment	Theory	Experiment	Theory	Experiment
0.17	0.17–0.19	0.243	0.28–0.31	0.32–0.346	0.34–0.50	0.43–0.51	0.51–0.70

^{a} n = number of baffles.

the value

$$\frac{I_B^{**}}{I_s} = \frac{2(d/a)(w/a)[2-(w/a)][\frac{1}{3}(d/a)^2+(1+k^2)/4]}{(h/a)[\frac{1}{12}(h/a)^2+\frac{1}{4}]} \approx 0.097 \quad (14)$$

and an upper limit for the moment of inertia.

The effective moment of inertia ratio is therefore

$$I_e/I_s = (I_e/I_s)_{swb} + I_B^{**}/I_s + (I_e/I_s)_{bd} \approx 0.346 \quad (15)$$

For a container with five baffles, the effect of the two additional baffles has to be added by multiplying the effective moment ratio of one baffle by $[1 + 4(d/a)^2]$. It is therefore

$$\frac{I_e^{(5)}}{I_r} = \left(\frac{I_e}{I_r}\right)_{swb} + \left(\frac{I_e}{I_r}\right)_{b_1} + 2\left(\frac{I_e}{I_r}\right)_{bd} + 2\left(\frac{I_e}{I_r}\right)_{b_2d} \quad (16)$$

which yields here

$$I_e^{(5)}/I_r = 0.21 + 0.033 + 0.077 + 2(0.054) = 0.43 \quad (17)$$

Again this value is smaller than the measured one. Considering the liquid completely trapped between the baffles would yield a value

$$I_e^{(5)}/I_r = (I_e/I_r)_{swb}(I_{r_{r=b}}/I_{r_{r=a}}) + I_B^*/I_r \sim 0.51 \quad (18)$$

where the value for the trapped liquid has been obtained from

$$\frac{I_B^*}{I_r} = \frac{(w/a)[\frac{1}{12}(h/a)^2+(1+k^2)/4][2-(w/a)]}{[\frac{1}{12}(h/a)^2+\frac{1}{4}]} \approx 0.38. \quad (19)$$

The following table summarizes the results of theoretical approximations and the experiments (n is number of baffles).

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Reply by Authors to Helmut F. Bauer

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THE authors welcome Professor Bauer's comments, which should be added to his already long list of contributions in the study of the dynamic behavior of liquids in moving tanks.

Before making a detailed reply, we would like to correct two slight errata in our Note (1). First, the heading for Table 1 should read $n = 0, 1, 3, 5$ instead of $n = 0, 1, 2, 3$ as printed. Second, in the calculations presented below Eq. (5) of the Note, the numerical values should be corrected to read $I_{\text{liq}} = 167$ kg − cm², $I_{\text{rig}} = 331$ kg-cm², and $I_{\text{liq}}/I_{\text{rig}} \approx 0.51$. These corrections indicate that the approximate theory and experiments agree even better than originally implied.

Dr. Bauer has given a very plausible method of calculating the moment of inertia of the liquid in a baffled tank. The arguments he uses [which lead to Eq. (1)] for calculating the moment of the inertia when the liquid near the baffles can be considered as rigid are quite similar to the ones we presented in our Note for the same case. His further theoretical work, which permits the calculation of moment of inertia for tanks in which the baffles are not closely spaced, is indeed valuable and will be of considerable help to missile designers. We are especially glad to see the close correlation between his theory and our test results; this gives us further confidence that both his theory and our experiments are essentially correct.

Reference

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Comments on "Approximate Re-Entry Velocity and Heating Equations"

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GLOVER[1] developed approximate equations developed for an atmosphere in hydrostatic equilibrium describing the time of flight and the total convective heat input at the stagnation point. It is the intent of this comment to cast these equations in a slightly different form which will allow their evaluation from standard mathematical tables. The particular equations derived here will assume an exponential atmosphere; however, one will recognize the equations to be of the same form as for any atmosphere in hydrostatic equilibrium. The nomenclature used herein is consistent with standard re-entry literature.

The velocity history may be approximated by[2]:

$$V = V_E e^{-\alpha e^{-\beta y}} \quad (1)$$

where α is defined by

$$\alpha \equiv \rho_0 C_D A / 2\beta m \sin\phi \quad (2)$$

The equation governing the time of flight becomes

$$t - t_0 = \frac{1}{\beta V_E \sin\phi} \int_{x_0}^{x} \frac{e^x}{x}\, dx \quad (3)$$

where $x = \alpha e^{-\beta y}$. The integral in Eq. (3) may be evaluated in terms of the exponential integral tabulated in Ref. 3:

$$\int_{x_0}^{x} \frac{e^x}{x}\, dx = \int_{-\infty}^{x} \frac{e^x}{x}\, dx - \int_{-\infty}^{x_0} \frac{e^x}{x}\, dx = Ei(x) - Ei(x_0) \quad (4)$$

Making use of a series expansion for the exponential integral in Ref. 4, we may write

$$Ei(x_0) = \ln x_0 + \gamma + x_0 + (x_0^2/2 \cdot 2!) + \ldots \quad (5)$$

where γ is Mascheroni's or Euler's constant. Assuming that

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